

# 3. Lato Lato

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Start with the assignment

### 3. Lato Lato

Attach a ball to each end of a string and connect the center of the string to a pivot.

When the pivot oscillates along the vertical direction, the balls start to collide and oscillate with increasing amplitude.

Investigate the phenomenon.

Czech translation (we put some effort into it, might help understanding)

### 3. Lato Lato (Klik-Klak)

Připevněte ke každému konci nitě kuličku a připojte střed nitě ke kolíku.

Kmitá-li kolík ve svislém směru, začnou se kuličky srážet a kmitat s rostoucí amplitudou.

Prozkoumejte tento jev.

Clear structure in three sentences

1. Setup
2. Phenomenon
3. Assignment—characteristically open

Start with resources at <https://www.tmfcr.cz/>

esp. the kit there [https://kit.ilyam.org/FDD\\_2025\\_IYPT\\_Reference\\_kit.pdf](https://kit.ilyam.org/FDD_2025_IYPT_Reference_kit.pdf)

First youtube videos

Ideal: <https://www.youtube.com/watch?v=FLHftISLNHE>

so it is possible, but may have problems: [https://www.youtube.com/watch?v=x\\_jN6JGA4dY](https://www.youtube.com/watch?v=x_jN6JGA4dY)

Try it

Simple enough to be shown at the tournament itself

Then references—learn what is known; acknowledge in the presentation  
typically in English

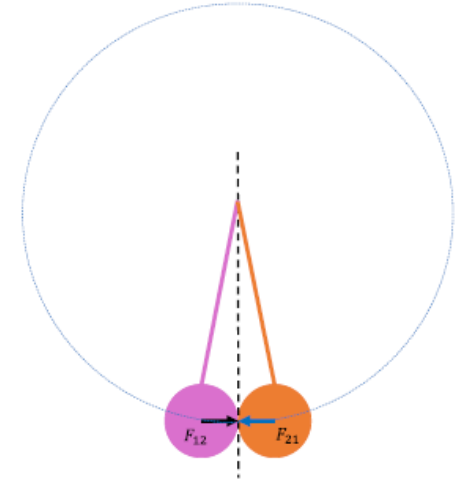
Some publicly available, for others you need help from somebody with access—I asked Hynek

Basic concepts and phenomena: relevant, not everything you know

(as they sometimes encourage in school, at least did in my school)

Collisions: elastic, inelastic

Conservation of momentum always, energy possibly



**Figure 3.** Illustration of the collision of two ball of Lato Lato 2.0 (front view). A black arrow heading to the right represents the action force ( $F_{12}$ ), and a blue arrow pointing to the left represents the reaction force ( $F_{21}$ ). The two arrows have the same length but point in different directions.

ref. from the kit:

E. Wibowo, N. Ulya, and P. Marwoto.

Demonstration of conservation of momentum using Lato Lato 2.0.

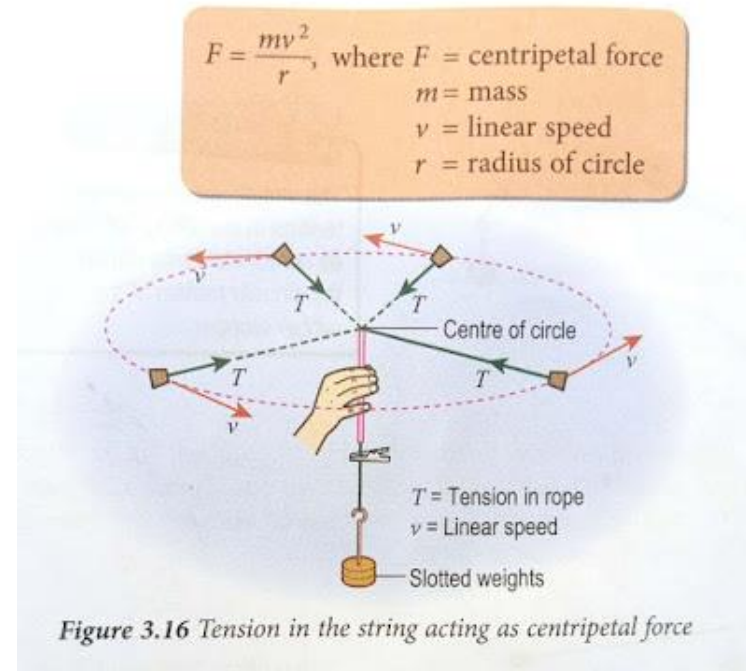
Phys. Educ. 59, 2, 025025 (2024)

Coefficient of restitution, [https://en.wikipedia.org/wiki/Coefficient\\_of\\_restitution](https://en.wikipedia.org/wiki/Coefficient_of_restitution)



Wikipedia a good source of general information  
at least in physics

## Centripetal acceleration



<http://cikgufazilah.blogspot.com/2020/07/centripetal-force-activity-physics-form.html>

Other internet sources might be OK too  
good sources of pictures

Condition for the taut string  $ml\dot{\theta}^2 + mg \cos \theta > 0$

ref. from the kit

F. Candra Funata and Z. Abidin. Playing lato-lato is difficult and this is why. Eur. J. Phys. 45, 4, 045007 (2024), [arXiv:2407.02951v1](https://arxiv.org/abs/2407.02951v1) [physics.class-ph]

The arXiv might have a free version of the paper

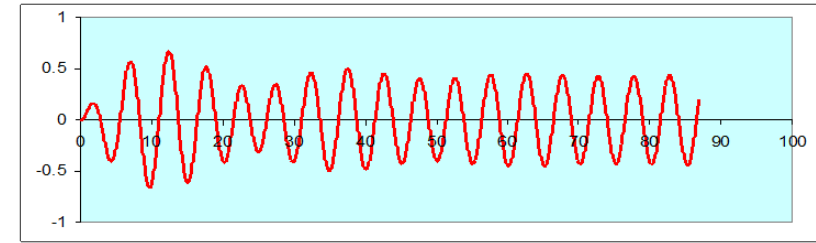
# Oscillations, damped and driven

simpler case: harmonic (ball on a spring)

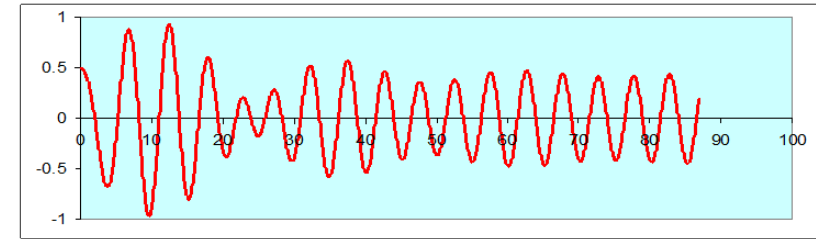
equation of motion

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 e^{i\omega t}$$

solution in terms of elementary functions



applet



Transients go away

<https://galileo.phys.virginia.edu/classes/152.mf1i.spring02/Oscillations4.htm>

College course lecture notes tend to be OK

Here un-harmonic oscillations

Driving mechanism: vertical force on the pivot has non-zero torque on the balls

increasing the angular momentum increasing the oscillations

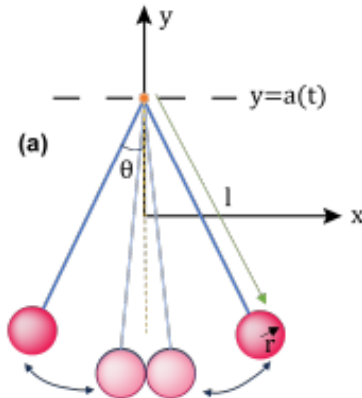
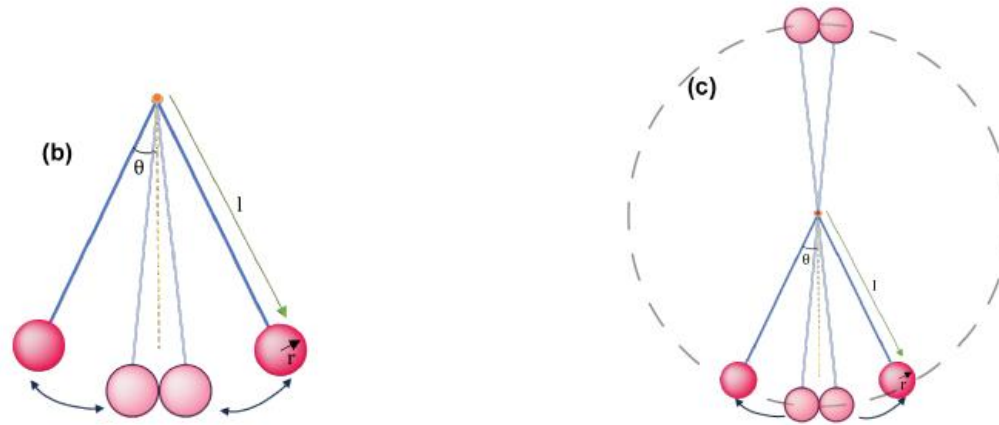


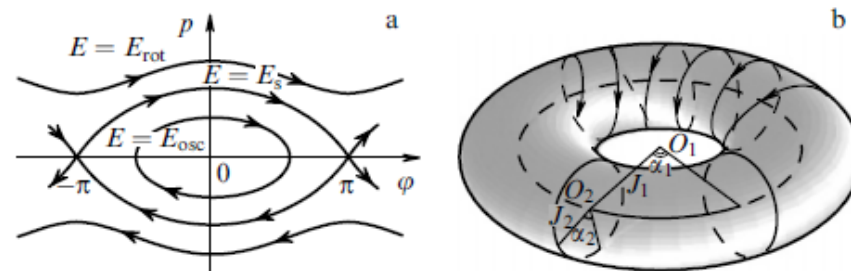
figure from the same Funata and Abidin paper

## Two phases



from the same Funata and Abidin paper

Two phases related to two phases of a pendulum: Lato-lato is like a pendulum in a mirror

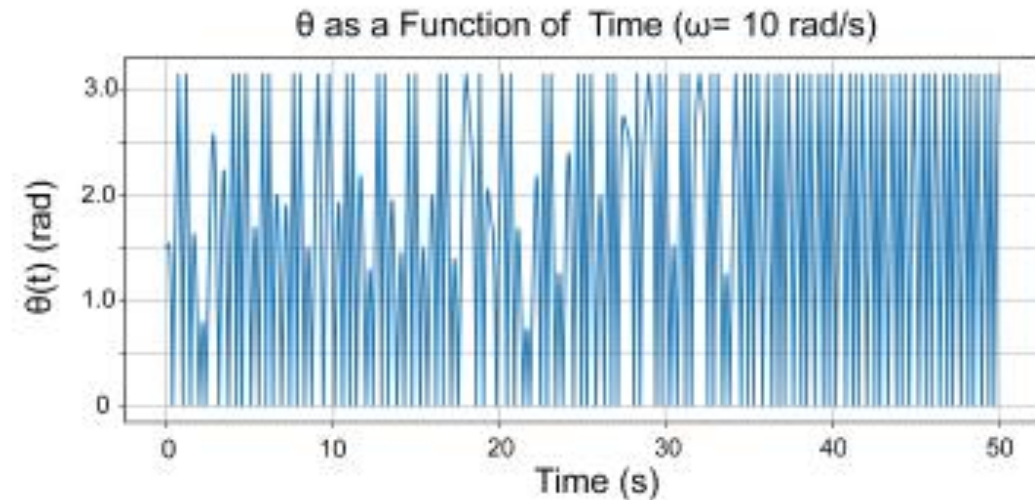
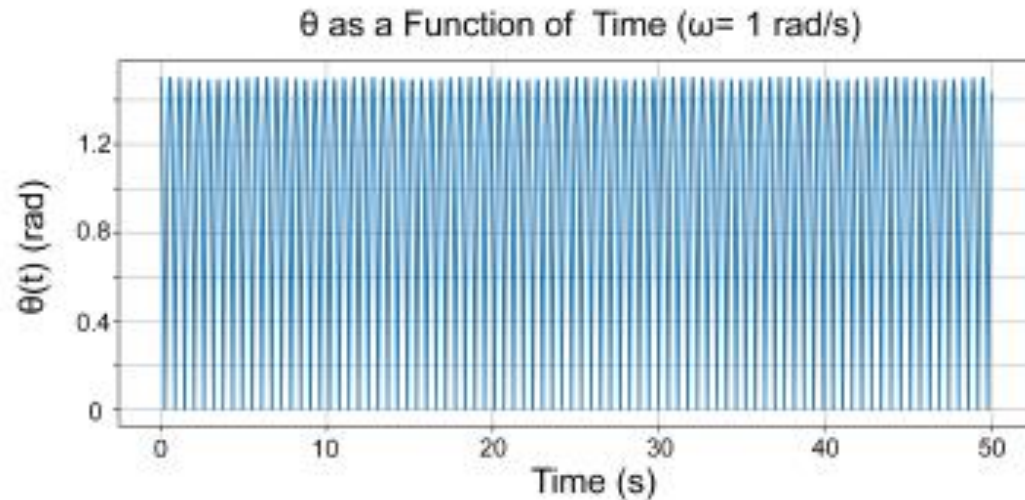


**Figure 1.** (a) Phase portrait of a nonlinear pendulum and (b) a visual representation of an integrable Hamiltonian system with two degrees of freedom in angle–action variables.

These were the basics

The title of the Funata and Abidin paper is Playing lato-lato is difficult and this is why, cf. the second video

→ More subtle aspect of the problem



Reference in the Funata and Abidin paper (even Hynek had to order it from the paper):

[9] Bartuccelli, M., Gentile, G., Georgiou, K.: Kam theory, lindstedt series and the stability of the upside-down pendulum. Mathematics 9 (2022) <https://doi.org/10.3934/dcds.2003.9.413>

They show the upside-down position of the a vertically driven pendulum is stable

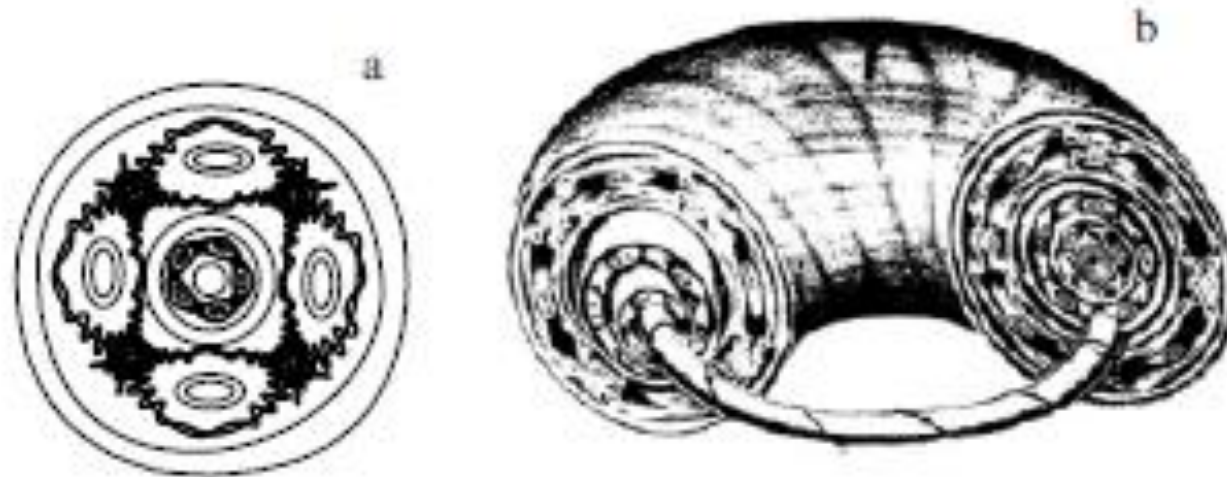


There might be chaos, a topic at the VYDRA spring symposium, at Hynek's suggestion

If not just planar motion, then we have 2 angles=2 degrees of freedom

At VYDRA I used the Loskutov paper on chaos as a resource

A figure from there showing chaos in a system with 2 degrees of freedom



**Figure 14.** Structure of the phase space of perturbed Hamiltonian systems with two degrees of freedom: (a) in the Poincaré map and (b) in the phase space [87].

Think of directions to explore